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NORMAL AND RECTIFYING CURVES IN GALILEAN SPACE G_3

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Abstract. In this study, we investigate normal and rectifying curve in the 3-dimensional Galilean space G_3 and give a parametrization for rectifying curves in this space.

Keywords: Normal curve, rectifying curve, Galilean space, Frenet formulas.

AMS Subject Classification: 53A35, 53A40.

1. Introduction

Galilean 3-space G_3 is simply defined as a Klein geometry of the product space RXE^2 whose symmetry group B_6 is Galilean transformation group which has an important place in classic and modern physics, for example, in quantum theory, gauge transformations in electromagnetism, in mecanics and conductivitiy tensors in fluid dynamics, also in mathematical fields such as Lagrangian mechanics, dynamics and control theory, and so on (see [1]).

A curve in Galilean 3-space G_3 is a graph of a plane motion. Note that such a curve is called a worldline in 3-dimensional Galilean space. It is well known that, the idea of worldlines originates in physics and was pioneered by Einstein. In physics, a world line of an object is the sequence of spacetime events corressponding to the history of the object. A world line is a special type of curve in spacetime. Each point of a world line is an event that can be labeled with the time and the spatial position of the object at that time. For example, the orbit of the Earth in space is approximately a circle, a three-dimensional curve in space. The Earth returns every year to the same point in space. However, it arrives there at a different time. The world line of the Earth is helical in spacetime and does not return to the same point.

The word line is now most often in relativity theories, i.e., general relativity and special relativity. The theory of special relativity puts some constraints on possible word lines. In special relativity the description of spacetime is limited to special coordinate systems that do not accelerate, called inertial coordinate systems. In such coordinate systems, the speed of light is a constant. Word lines of particles or objects at constant speed are called geodesics. The use of word lines in general relativity is basically the same as in special relativity with the difference that spacetime can be curved. A metric exists and its dynamics are determined by the Einstein field equations and are depended on the mass distribution in spacetime.

From the differential geometric point of view, the study of curves in G_3 has its own interest. Many interesting results on curves in G_3 have been obtained by many authors (see [4],[9]-[12]).

In this study, other important subject is normal and rectifying curves in Galilean 3-space G_3. In the Euclidean 3-space, rectifying curves are introduced by B.Y. Chen in [2] as space curves whose position vectors always lies in its rectifying plane, spanned by the tangent and the binormal vector fields of the curve. Rectifying curves and normal curves in Euclidean space and Minkowski space are studied in [2],[5], [6]. A relationship between the rectifying curves and the centrodes, which play some important roles in mechanics, kinematics as well as in defining the curve of constant precession.

The literature survey indicated that, there is no normal and rectifying curves in Galilean 3-space. Thus, the study is proposed to serve such a need. In this paper, making use of method in [2] and [5], we define the normal curve and rectifying curve in Galilean 3-space G_3 and characterize normal and rectifying curves lying fully in G_3. In particular, we prove that the ratio of torsion and curvature of any rectifying curve in G_3 is nonconstant linear function of the invariant parameter x. Also we obtain a parametrization of rectifying curves lying fully in the Galilean 3-space.

2. Preliminaries

Let $E^2 = \left(R^2(y,z), dy^2 + dz^2\right)$ be Euclidean plane. The Galilean 3-space is a product space

$$G_3 = R(x) \times E^2(y,z)$$

with Galilean group B_6:

$$B_6 = \{ f = f(v,\varphi) : G_3 \to G_3 \},$$

where

$$f(v,\varphi)\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ v^y & \cos\varphi & \sin\varphi \\ v^z & \sin\varphi & \cos\varphi \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} a \\ b \\ c \end{pmatrix}.$$

The Galilean group B_6 is generated by Euclidean motion group $E(2)$ and constant velocity motions:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \to \begin{pmatrix} x \\ y + v^y x \\ z + v^z x \end{pmatrix}.$$

An element $f \in B_6$ is called a Galilei transformation.

In G_3 there are four classes of lines:

a) (proper) nonisotropic lines - they do not meet the absolute line f .

b) (proper) isotropic lines - lines that do not belong to the plane w but meet the absolute line f.

c) unproper nonisotropic lines - all lines of w but f.

d) the absolute line f.

Planes $x = const.$ are Euclidean and so is the plane w. Other planes are isotropic, [8].

For a curve $c : I \to G_3$, $I \subseteq R$ parametrized by the invariant parameter $s = x$, is given in the coordinate form

$$c(x) = (x, y(x), z(x)). \tag{1}$$

The curvature $\kappa(x)$ and the torsion $\tau(x)$ are defined by

$$\kappa(x) = \sqrt{y''(x)^2 + z''(x)^2}, \tau(x) = \frac{det(c'(x), c''(x), c'''(x))}{\kappa^2(x)} \tag{2}$$

The associated moving trihedron is given by

$$\begin{aligned} T(x) &= c'(x) = (1, y'(x), z'(x)), \\ N(x) &= \frac{1}{\kappa(x)} c''(x) = \frac{1}{\kappa(x)}(0, y''(x), z''(x)), \\ B(x) &= \frac{1}{\kappa(x)}(0, -z''(x), y''(x)). \end{aligned} \tag{3}$$

In Affine coordinates, the Galilean scalar product between the points $P_i = (x_i, y_i, z_i),\ i = 1,2$ is defined by

$$\langle P_1, P_2 \rangle_g = \begin{cases} |x_2 - x_1|, & if\ \ x_1 \neq x_2 \\ (y_2 - y_1)^2 + (z_2 - z_1)^2, & if\ \ x_1 = x_2 \end{cases}.$$

The Galilean cross product is defined for $\mathbf{a} = (a_1, a_2, a_3)$, $\mathbf{b} = (b_1, b_2, b_3)$ by [12]

$$a \times_g b = \begin{vmatrix} 0 & e_1 & e_2 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix},$$

One can see that

$$\mathsf{F}(x)=(T(x),N(x),B(x))=\begin{pmatrix} 1 & 0 & 0 \\ y'(x) & \dfrac{y''(x)}{\kappa(x)} & \dfrac{-z''(x)}{\kappa(x)} \\ z'(x) & \dfrac{z''(x)}{\kappa(x)} & \dfrac{y''(x)}{\kappa(x)} \end{pmatrix} \tag{4}$$

takes value in B_6. This mapping $\mathsf{F}(x)$ is called the Galilei frame of $c(s)$. The Galilei frame satisfies the following Frenet equations:

$$\frac{d}{dx}\mathsf{F}=\mathsf{F}\begin{pmatrix} 0 & 0 & 0 \\ \kappa & 0 & -\tau \\ 0 & \tau & 0 \end{pmatrix}. \tag{5}$$

From formulas (5), we have the following relation [11]

$$c'''(x)=\kappa(x)\tau(x)B(x)+\kappa(x)N(x) \tag{6}$$

The vectors T,N and B are called the tangent, the principal normal and the binormal vector of c, respectively.

The planes spanned by the vector $\{T,N\}$, $\{N,B\}$, $\{T,B\}$ are called the osculating plane, the normal plane and the rectifiying plane.

Galilean sphere of radius 1 and center at the origin is defined by

$$S^2(1)=\{v\in G_3 \mid \langle v,v\rangle_g=1\},$$

[4].

3. Some characterizations of normal and rectifying curves in G_3

In this section, we define the normal curve and rectifying curve in G_3 and give some characterizations for normal curves and rectifying curves lying fully in the Galilean 3-space.

Definition 1. Let $c=c(x)$ be a curve in 3-dimensional Galilean space G_3. If the position vector of c always lies in its normal plane then it is called normal curve in G_3.

By definition, for a curve in G_3, the position vector c satisfies

$$c(x)=\xi(x)N(x)+\eta(x)B(x),$$

where x is a Galilean invariant-the arc length on c and $\xi(x),\eta(x)$ are differentiable functions

Theorem 1. Let $c = c(x)$ be a normal curve in G_3 with constant curvature $\kappa > 0$ and nonzero constant torsion τ. Then c is a normal curve if and only if the principal normal and binormal components of the position vector c are given by

$$
\begin{aligned}
< c, N >_g = \xi(x) = &-\frac{1}{4}\kappa x^2 e^{-2i\tau x}(e^{2i\tau x}-1)^2 + \frac{1}{4}\kappa x^2 (e^{i\tau x}+e^{-i\tau x})^2 \\
&-\frac{\kappa i e^{-2i\tau x}(-i+\tau x+e^{2i\tau x}(i+\tau x))(-1-i\tau x+e^{2i\tau x}(1-i\tau x))}{4\tau^2} \\
&-\frac{\kappa e^{-2i\tau x}(1+i\tau x+e^{2i\tau x}(1-i\tau x))^2}{4\tau^2} + \frac{1}{2}C_1 x e^{-i\tau x}(1+e^{2i\tau x}) \\
&-\frac{1}{2}C_2 i x e^{-i\tau x}(-1+e^{2i\tau x}) + \frac{1}{2}C_3 e^{-i\tau x}(1+i\tau x+e^{2i\tau x}(1-i\tau x)) \\
&-\frac{1}{2}C_4 e^{-i\tau x}(-i+\tau x+e^{2i\tau x}(i+\tau x))
\end{aligned}
$$

and

$$
\begin{aligned}
< c, B >_g = \eta(x) = &\frac{i\kappa e^{-2i\tau x}(1+i\tau x+e^{2i\tau x}(1-i\tau x))(-1-i\tau x+e^{2i\tau x}(1-i\tau x)}{4\tau^2} \\
&-\frac{\kappa e^{-2i\tau x}(-i+\tau x+e^{2i\tau x}(i+\tau x))(1+i\tau x+e^{2i\tau x}(1-i\tau x))}{4\tau^2} \\
&+\frac{1}{2}C_1 i x(e^{i\tau x}-e^{-i\tau x}) + \frac{1}{2}C_2 x(e^{i\tau x}+e^{-i\tau x}) \\
&+\frac{1}{2}C_3 e^{-i\tau x}(-i+\tau x+e^{2i\tau x}(i+\tau x)) + \frac{1}{2}C_4 e^{-i\tau x}(1+i\tau x+e^{2i\tau x}(1-i\tau x)),
\end{aligned}
$$

respectively, where C_1, C_2, C_3 and C_4 are constants.

Proof. Let us suppose that $c(x)$ is a normal curve, then from Definition 1, we have

$$c(x) = \xi(x)N(x) + \eta(x)B(x).$$

By taking the derivative of this equation with respect to x twice and using the Frenet equations (2.5), we get following linear differential equation system

$$
\begin{aligned}
&\xi'' - 2\tau\eta' - \tau^2\xi = \kappa \\
&\eta'' + 2\tau\xi' - \tau^2\eta = 0.
\end{aligned}
$$

By solving this system, we obtain

$$\xi(x) = -\frac{1}{4}\kappa x^2 e^{-2i\tau x}(e^{2i\tau x}-1)^2 + \frac{1}{4}\kappa x^2(e^{i\tau x}+e^{-i\tau x})^2$$

$$-\frac{\kappa i e^{-2i\tau x}(-i+\tau x+e^{2i\tau x}(i+\tau x))(-1-i\tau x+e^{2i\tau x}(1-i\tau x))}{4\tau^2}$$
$$-\frac{\kappa e^{-2i\tau x}(1+i\tau x+e^{2i\tau x}(1-i\tau x))^2}{4\tau^2}+\frac{1}{2}C_1 x e^{-i\tau x}(1+e^{2i\tau x})$$
$$-\frac{1}{2}C_2 i x e^{-i\tau x}(-1+e^{2i\tau x})+\frac{1}{2}C_3 e^{-i\tau x}(1+i\tau x+e^{2i\tau x}(1-i\tau x))$$
$$-\frac{1}{2}C_4 e^{-i\tau x}(-i+\tau x+e^{2i\tau x}(i+\tau x))$$

and

$$\eta(x)=\frac{i\kappa e^{-2i\tau x}(1+i\tau x+e^{2i\tau x}(1-i\tau x))(-1-i\tau x+e^{2i\tau x}(1-i\tau x)}{4\tau^2}$$
$$-\frac{\kappa e^{-2i\tau x}(-i+\tau x+e^{2i\tau x}(i+\tau x))(1+i\tau x+e^{2i\tau x}(1-i\tau x))}{4\tau^2}$$
$$+\frac{1}{2}C_1 i x(e^{i\tau x}-e^{-i\tau x})+\frac{1}{2}C_2 x(e^{i\tau x}+e^{-i\tau x})$$
$$+\frac{1}{2}C_3 e^{-i\tau x}(-i+\tau x+e^{2i\tau x}(i+\tau x))+\frac{1}{2}C_4 e^{-i\tau x}(1+i\tau x+e^{2i\tau x}(1-i\tau x)),$$

respectively, where C_1, C_2, C_3 and C_4 are constants, which completes the proof.

Definition 2. Let c be a curve in 3-dimensional Galilean space G_3. If the position vector of c always lies in its rectifying plane then it is called rectifying curve in G_3.

Theorem 2. Let $c=c(x)$ be a rectifying curve in G_3, the curvature $\kappa(x)>0$. Then the following statements hold:

(i) The distance function $\rho=\|c\|$ satisfies

$$\rho^2=\left|x^2+mx+n\right|,$$

for some $m\in R,\ n\in R-\{0\}$.

(ii) The tangential component of the position vector of c is given by

$$\langle c,T\rangle_g=x+m_1,$$

where $m_1\in R$.

(iii) The normal component c^N of the position vector of the curve has a constant length.

(iv) The binormal component of the position vector of the curve c is constant.

Conversly, if $c = c(x)$ is a curve in G_3 and one of the statements (i), (ii), and (iii), (iv) holds, then c is a rectifying curve.

Proof. Let us assume that $c = c(x)$ is a rectifying curve in G_3. Then from Definition 1., we can write the position vector of c by

$$c(x) = \lambda(x)T(x) + \mu(x)B(x), \tag{7}$$

where $\lambda(x)$ and $\mu(x)$ are some differentiable functions of the invariant parameter x.

i) Differentiating the equation (7) with respect to x and considering the Frenet equations (5) we get

$$\lambda'(x) = 1, \quad \lambda(x)\kappa(x) - \mu(x)\tau(x) = 0, \quad \mu'(x) = 0. \tag{8}$$

Thus, we obtain

$$\begin{gathered} \lambda(x) = x + m_1, m_1 \in R \\ \mu(x) = n_1, n_1 \in R \\ \mu(x)\tau(x) = \lambda(x)\kappa(x) \neq 0, \end{gathered} \tag{9}$$

and hence

$$\mu(x) = n_1 \neq 0, \tau(x) \neq 0.$$

From the equation (7), we easily find that

$$\rho^2 = \left|\langle c, c\rangle_g\right| = \left|x^2 + mx + n\right|,$$

$m \in R$, $n \in R - \{0\}$. ii) If we consider equation (7), we get

$$\langle c, T\rangle_g = \lambda(x),$$

which means that the tangential component of the position vector of c is given by $\langle c, T\rangle_g = x + m_1$, where $m_1 \in R$.

iii) From the equation (7), it follows that the binormal component c^N of the position vector c is given by

$$c^N = \mu B$$

and we get $\left\|c^N\right\|_g = |\mu| \neq 0$.

iv) If we consider equation (7) we easily get $\langle c, B\rangle_g = \mu = const.$

Conversely, suppose that statement (i) or statement (ii) holds. Then we have

$$\langle c(x), T(x)\rangle_g = x + m_1, m_1 \in R. \tag{10}$$

Differentiating equation (10) with respect to x, we obtain

$$\kappa\langle c, N\rangle_g = 0.$$

Since $\kappa(x) > 0$, it follows that

$$\langle c(x), N(x)\rangle_g = 0,$$

which means that c is a rectifying curve. Next, suppose that statement (iii) holds. Let us write

$$c(x) = l(x)T(x) + c^N, l(x) \in R.$$

Then we easily obtain that

$$\langle c^N, c^N\rangle_g = K = constant = \langle c, c\rangle_g - \langle c, T\rangle_g^2 \tag{11}$$

If we differentiate equation (3.5) with respect to $x,$ we get

$$\langle c, T\rangle_g = \langle c, T\rangle_g \left(1 + \kappa\langle c, N\rangle_g\right). \tag{12}$$

Since $\rho \neq const.$, we have

$$\langle c, T\rangle_g \neq 0.$$

Morever , since $\kappa(x) > 0$ and from (3.6) we obtain

$$\langle c, N\rangle_g = 0,$$

that is c is a rectifying curve.

Finally, if the statement (iv) holds, then from the Frenet equations (5), we get

$$\langle c, N\rangle_g = 0,$$

which means that c is a rectifying curve.

Theorem 3. Let $c = c(x)$ be a curve in G_3. Then up to isometries of G_3, the curve c is a rectifying if and only if there holds

$$\frac{\tau(x)}{\kappa(x)} = ax + b,$$

where $a \in R - \{0\},\ b \in R$.

Proof. Let us first suppose that the curve $c(x)$ is rectifying. From the equations (8) and (9) we easily find that

$$\frac{\tau(x)}{\kappa(x)} = ax + b, a \in R - \{0\}, b \in R.$$

Conversely, let us suppose that

$$\frac{\tau(x)}{\kappa(x)} = ax + b, a \in R - \{0\}, b \in R.$$

Then we may choose

$$a = \frac{1}{n_1}, b = \frac{m_1}{n_1}, n_1 \in R - \{0\}, m_1 \in R.$$

Thus we have

$$\frac{\tau(x)}{\kappa(x)} = \frac{x + m_1}{n_1}.$$

If we consider the Frenet equations (5), we easily find that

$$\frac{d}{dx}\left(c(x) - (x + m_1)T(x) - n_1 B(x)\right) = 0,$$

which means that c is a rectifying curve.

Theorem 4. Let $c = c(x)$ be a curve in G_3. Then c is a rectifying curve if and only if, up to a parametrization, c is given by

$$c(t) = \alpha(t)\frac{|n_1|}{\cos t}, n_1 \in R,$$

where $\alpha(t)$ is a curve lying in the Galilean sphere $S^2(1)$.

Proof. Let us suppose that c is a rectifying curve in G_3. By Theorem 2. ,we have

$$\rho^2 = \|c\|_g^2 = (x + m_1)^2 + n_1^2, m_1 \in R, n_1 \in R - \{0\}.$$

Also, we may apply a translation with respect to x, such that $\rho^2 = x^2 + n_1^2$. Now, let define a curve $\alpha(x)$ lying in the Galilean sphere $S^2(1)$ by

$$\alpha(x) = \frac{c(x)}{\rho(x)}. \tag{13}$$

Then we have

$$c(x) = \alpha(x)\sqrt{x^2 + n_1^2}. \tag{14}$$

If we differentiate (14) with respect to x, we obtain

$$T(x) = \alpha(x)\frac{x}{\sqrt{x^2 + n_1^2}} + \alpha'(x)\sqrt{x^2 + n_1^2}. \tag{15}$$

Since $\langle \alpha, \alpha \rangle_g = 1$, we have $\langle \alpha, \alpha' \rangle_g = 0$. From (15) we get

$$1 = \langle T, T \rangle_g = \langle y', y' \rangle_g \left(x^2 + n_1^2\right) + \frac{x^2}{x^2 + n_1^2},$$

and hence

$$\langle y', y' \rangle_g = \frac{n_1^2}{(x^2 + n_1^2)^2}. \tag{16}$$

From (16) we obtain

$$\left\| y'(x) \right\| = \frac{|n_1|}{x^2 + n_1^2}.$$

Let define the parameter of the curve y by

$$t = \int_0^x \left\| \alpha'(u) \right\|_g du.$$

Then we have

$$t = \int_0^x \frac{|n_1|}{u^2 + n_1^2} du$$

and therefore

$$x = |n_1| \tan x. \tag{17}$$

Substituting (17) into (18) we obtain the parametrization

$$c(t) = \alpha(t) \frac{|n_1|}{\cos t}, n_1 \in R \tag{18}$$

Conversely, suppose that c is a curve defined by (18) where $\alpha(t)$ is a curve lying in the Galilean sphere $S^2(1)$. Differentiating the equation (18) with respect to t, we get

$$c'(t) = \frac{|n_1|}{\cos^2 t} (\alpha(t) \sin t + \alpha'(t) \cos t).$$

By assumption we have $\langle \alpha', \alpha' \rangle_g = 1$, $\langle \alpha, \alpha \rangle_g = 1$ and $\langle \alpha, \alpha' \rangle_g = 0$. Thus it follows that

$$\langle c, c' \rangle_g = \frac{n_1^2 \sin t}{\cos^3 t}, \langle c', c' \rangle_g = \frac{n_1^2}{\cos^4 t} \quad and \quad \|c'\|_g = \frac{|n_1|}{\cos^2 t}.$$

Let us put

$$c(t) = l(t) c'(t) + c^N,$$

where $l(t) \in R$ and c^N is a normal component of the position vector c. Then we have easily find that

$$l = \frac{\langle c, c' \rangle_g}{\langle c', c' \rangle_g}$$

and

$$\langle c^N, c^N \rangle_g = \langle c, c \rangle_g - \frac{\langle c, c' \rangle_g^2}{\langle c', c' \rangle_g}. \tag{19}$$

If we consider the relation $\langle c, c\rangle_g = \dfrac{n_1^2}{cos^2 t}$ in (19) we get

$$\langle c^N, c^N\rangle_g = n_1^2 = const.$$

and $\left\|c^N\right\| = const.$ and since $\rho = \|c\|_g = \left|\dfrac{n_1}{cos\,t}\right| \neq const.$ Thus, from Theorem 4. implies that c is a rectifying curve in G_3.

Example1. Consider a curve $c = c(t)$ in G_3

$$c(t) = (0, \frac{sin\,2t}{cos\,t}, \frac{cos\,2t}{cos\,t}),$$

where $t = \int_0^x \frac{1}{u^2+1}du.$ Then c is a rectifying curve in Galilean 3-space.

Example 2. The curve $c = c(x)$ in G_3 defined by

$$c(x) = (0, 3e^x + 3e^{-x} - 10e^{(\sqrt{2}-1)x} - 10e^{(-\sqrt{2}-1)x} - 1, 10e^{(\sqrt{2}-1)x} + 10e^{(-\sqrt{2}-1)x})$$

is a normal curve in Galilean 3-space.

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Qaliley fəzasında normal və düzələn əyrilər

Handan Öztəkin

XÜLASƏ

Bu məqalədə biz Qaliley fəzasında normal və düzələn əyriləri öyrənir və bu fəzada düzələn əyrilərin parametrizasiyasını veririk.

Açar sözlər: normal əyri, düzələn əyri, Qaliley fəzasıç Frenet düsturu.

Нормальные и выпрямляемые кривые в пространстве Галилея

Хандан Озтекин

РЕЗЮМЕ

В статье исследуются нормальные и выпрямляемые кривые в пространстве Галилея и дается параметризация выпрямляемых кривых в этом пространстве.

Ключевые слова: нормальная кривая, выпрямляемая кривая, пространство Галилея, формулы Френета